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Semi-Lagrangian Lattice Boltzmann Method for Compressible Flows

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Introduction

In the field of weakly compressible and isothermal flows, the lattice Boltzmann method (LBM) is an established tool for Computational Fluid Dynamics. However, in the field of compressible flows, there is no generally accepted framework. In addition, Eulerian solvers like finite difference or finite volume LBM suffer from high computational costs.

We present an extension of the semi-Lagrangian lattice Boltzmann method (SLLBM) for compressible flows, which is based on a cell-based interpolation of the simulation domain.

Key features

- *No time integrator needed*
- *Adjustable time step size*
- *Spatially high-order solution*
- *Unstructured meshes supported*

Methodology

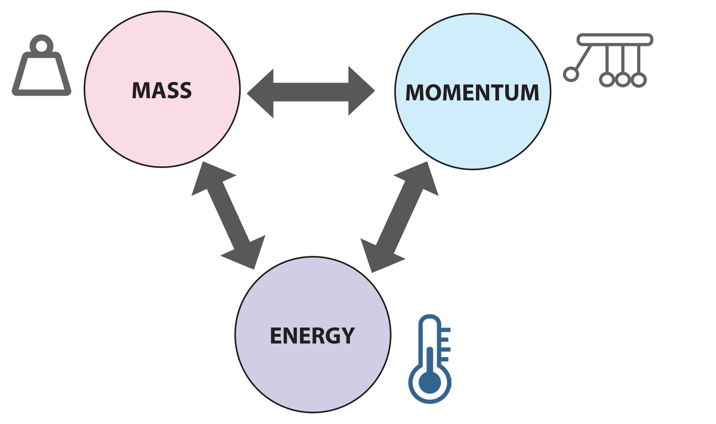
$$f_i(x,t) = f_i(x-\delta_t\xi_i, t-\delta_t) - \frac{1}{\tau}\left[f_i(x-\delta_t\xi_i, t-\delta_t) - f_i^{\mathrm{eq}}(x-\delta_t\xi_i, t-\delta_t)\right]$$

The lattice Boltzmann method solves the Navier-Stokes equations by a stream and collide algorithm of the particle distribution function f. Instead of the node-to-node streaming step, the Semi-Lagrangian lattice Boltzmann method determines the departure point by interpolation.

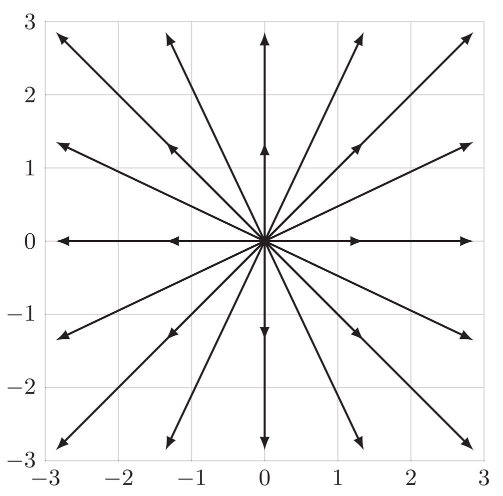
$$\rho = \sum_i f_i$$

$$\rho u = \sum_i \xi_i f_i$$

$$2\rho E = \sum_i \xi_i^2 f_i$$



I. D2Q25 velocity set



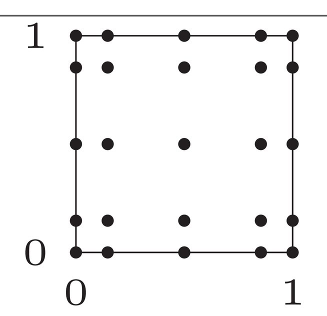
Velocity set based on the roots of the 5th order Hermite polynomials

i	ξ_i	w_i
0	0	8/15
1, 3	$\pm\sqrt{5-\sqrt{10}}$	$7+2\sqrt{10}/60$
2, 4	$\pm\sqrt{5+\sqrt{10}}$	$7-2\sqrt{10}/60$

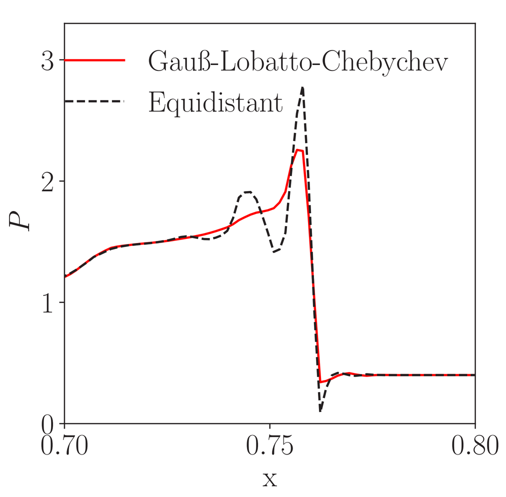
Particle velocities and weights of the underlying D1Q5

II. Cell-based interpolation

- Interpolation up to fourth order
- Use of Gauß-Lobatto-Chebyshev support points to minimize oscillations



Distribution of support points in the reference cell.



Comparison of the pressure lines in the 2D Riemann problem for equidistant and Gauß-Lobatto-Chebyshev nodes. The latter allows for stable simulations.

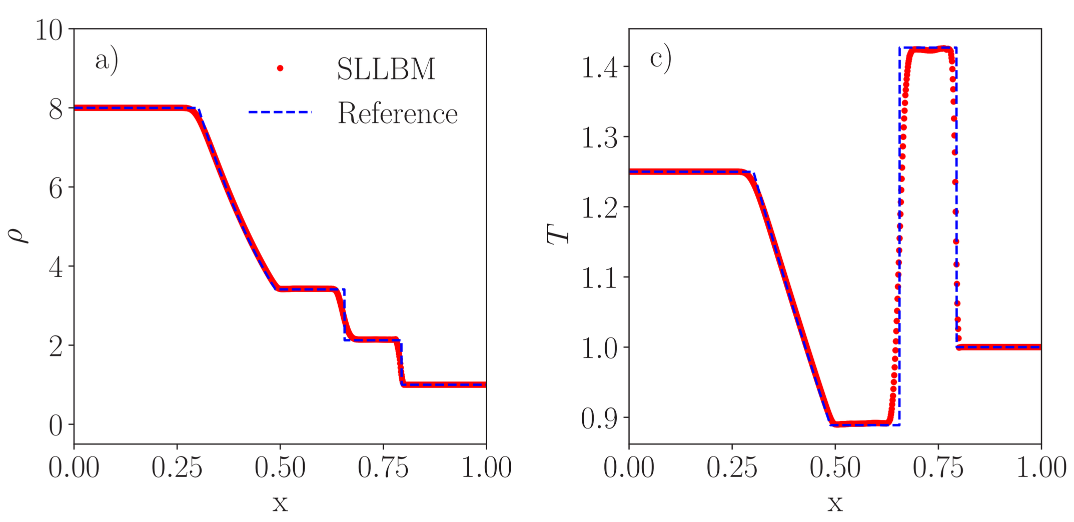
III. Equilibrium by Hermite projection

Expansion up to fourth order enables compressible flows for LBM

$$f^{\mathrm{eq},N}(\hat{\xi}) \approx \omega(\hat{\xi}) \sum_{n=0}^{N} \frac{1}{n!} a^{(n)}(\mathbf{x},t) \cdot \mathcal{H}^{(n)}(\hat{\xi})$$

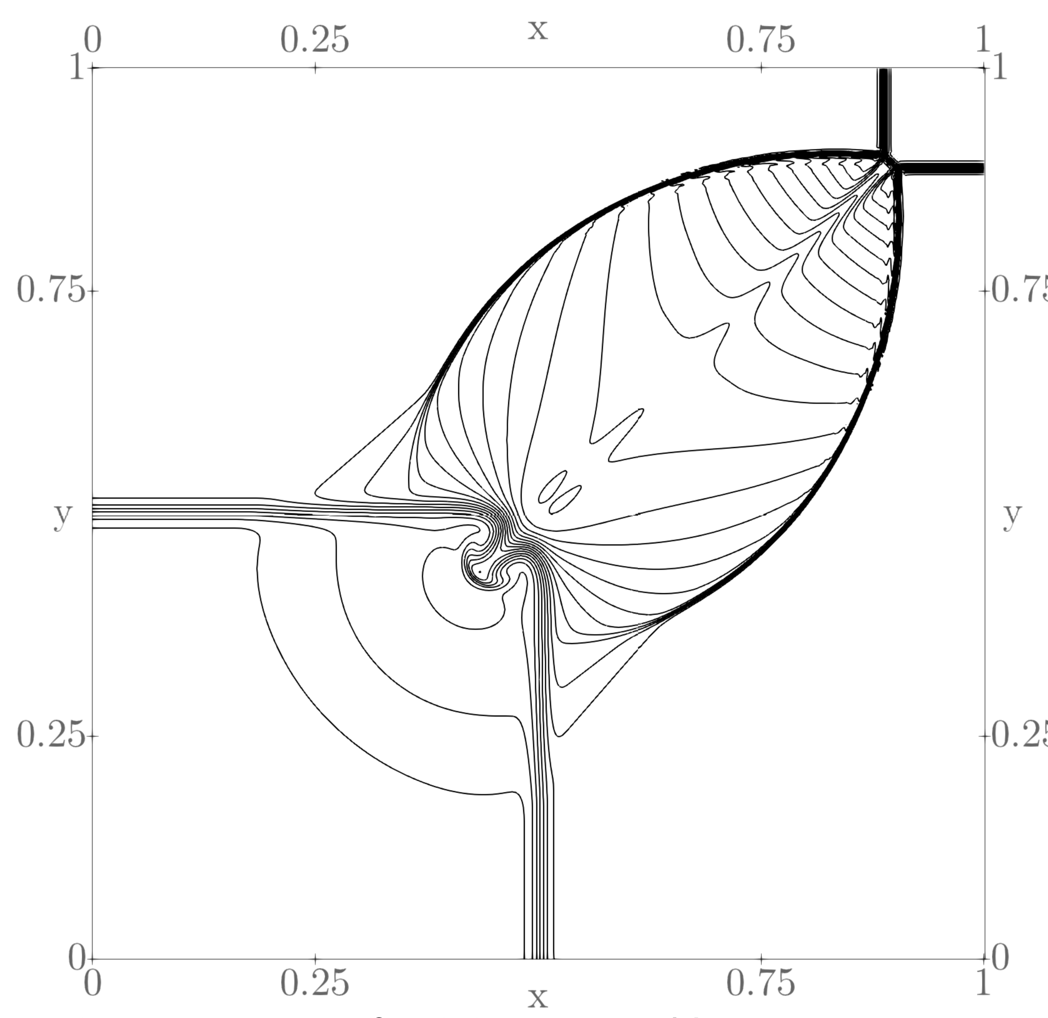
Results

A. Sod shock tube



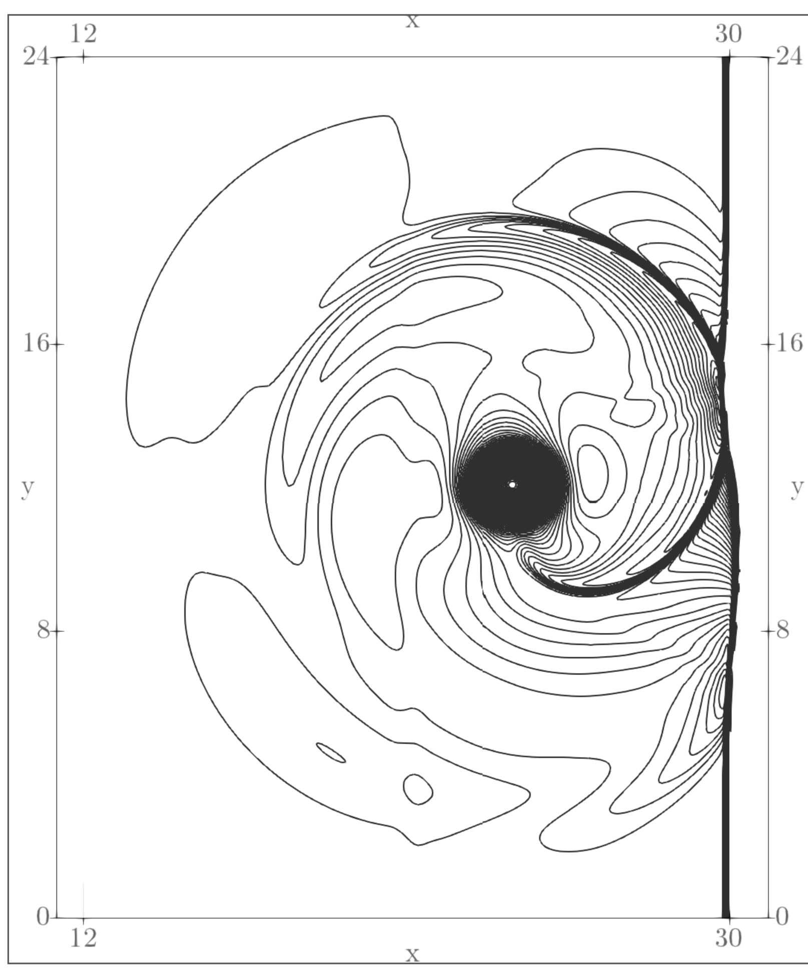
- Density and temperature lines in comparison to the reference
- 200 grid points in 100 cells
- order of interpolation polynomial: 2

B. 2D Riemann problem



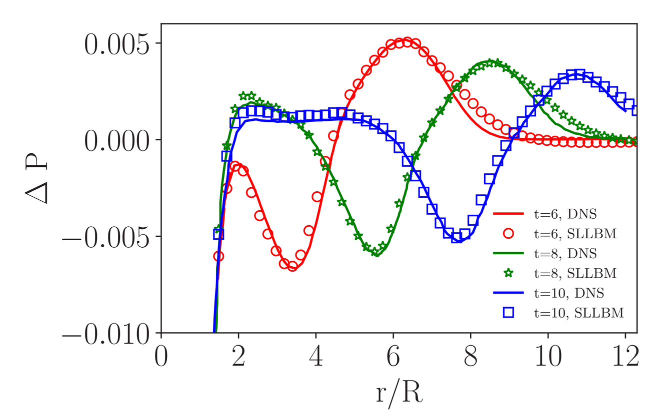
- Density contours of a 2D Riemann problem
- 512 x 512 grid points with 128 x 128 cells
- order of interpolation polynomial: 4

C. Shock-vortex interaction

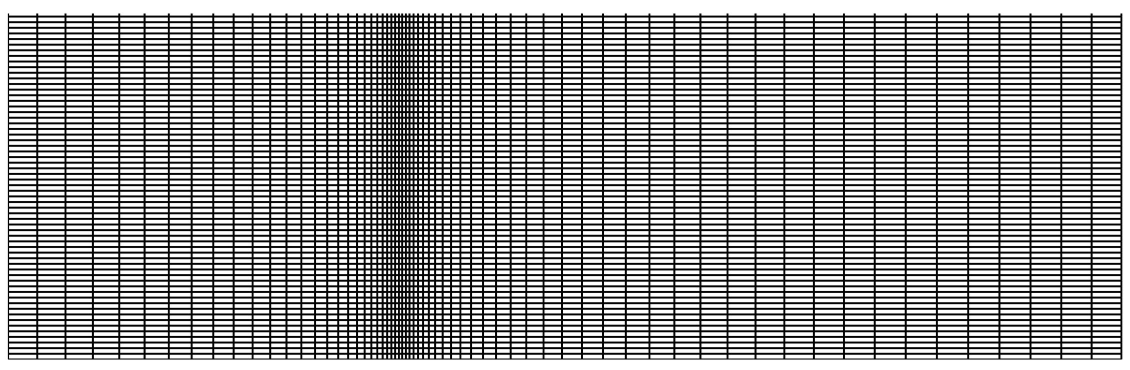


Density contours of a shock-vortex interaction as in O. Inoue and Y. Hattori, J. Fluid Mech. 380, 81 (1999).

- 512x512 grid points
- Order of interpolation polynomial: 4
- Medium resolution of normalized radius R: 7.2 points in x-direction.



Evaluation of the sound pressure curves from the center of the vortex after times 6, 8, 10 compared to the reference by Inoue and Hattori.



In contrast to standard LBM, the SLLBM allows for stretched grids, which were used in the shock-vortex simulations.

Article

Submitted to Phys. Rev. E

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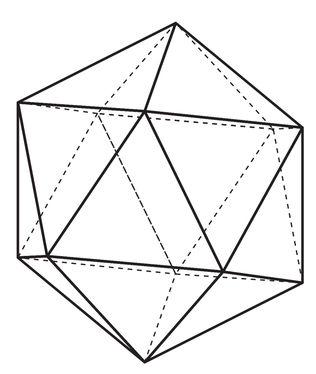
Outlook



- Extension to 3 dimensions
- Test of new velocity sets
- Application to viscous test cases with solid boundaries